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FINAL REPORT ON CONTRACT NO. DAAL-03-87-K-0022

(ARO project number: P-24025-EG)

Title of Project

**UNSTEADY FREE-WAKE VISCOUS AERODYNAMIC
ANALYSIS OF HELICOPTER ROTORS**

Authors of report

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Boston University**

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February 9, 1987 to October 8, 1989

1 Introduction

This is the final report on Contract No. DAAL-03-87-K-0022 (ARO project number: P-24025-EG) for research on unsteady free-wake viscous aerodynamic analysis of helicopter rotors. The effort accomplished under the contract may be divided into three general areas. The first deals with further developments of the zeroth-order potential-flow analysis; the second with the first-order analysis; and the third with the Poincaré decomposition for the analysis of viscous flows. For the sake of clarity, each of these areas is reported separately in the following sections. Developments that have been reported in preceding Semiannual Reports are briefly outlined, whereas the more recent developments are discussed in more detail.

2 Zeroth-order potential-flow formulation

The zeroth-order potential-flow formulation, developed under a preceding contract, was presented in Ref. 1. This work was summarized as a journal paper and published in Vertica (Morino and Bharadvaj, 1988, Ref. 2). Also the formulation was extended to compressible flows by Morino, Freedman, and Tseng (1987, Ref. 3). A paper that combines the compressible-flow formulation and the free-wake formulation was presented at a IUTAM Symposium (Morino, Bharadvaj, Freedman, and Tseng, 1987, Ref. 4); this includes comparisons with the experimental results of Caradonna and Tung (Ref. 5). A slightly revised version of the paper was recently published in the journal Computational Mechanics (Morino, Bharadvaj, Freedman, and Tseng, 1987, Ref. 6). An improvement in the comparison with the results of Caradonna and Tung was obtained by Affes (1987, Ref. 7) in his master thesis; his results were obtained with a free-wake iterative scheme (not time-accurate) that maintains its stability as the number of elements in the radial direction increases. Finally, a review paper has been prepared that summarizes much of the activity described above (Morino and Tseng, Ref. 8; this will appear as chapter of a book that will be published shortly).

Finally, recent results have been presented at the International Symposium on Boundary Element Methods held in East Hartford in October 1989 (Refs. 9 and 10). Gennaretti and Morino (Ref. 9) present an improved algorithm for compressible flows around helicopter rotors (which is valid for higher values of the tip Mach number, even at $M_{tip} = .72$). Their results include the evaluation of the pressure distribution which are in very good agreement with the experimental results of Caradonna and Tung (Ref. 5) and demonstrate that the compressibility effects are captured correctly. Incidentally, applications to aeroacoustics of propellers are also included in Ref. 9. Mastroddi and Morino (Ref. 10), include applications of the methodology to the evaluation of the aerodynamic forces that occur in flutter. A comparison indicates that our more sophisticated approach yields results that are considerably different from those obtained with the classical Loewy's formulation.

3 First-order potential-flow formulation

A method for free-wake analysis of rotors was introduced by Morino and Bharadvaj (Ref. 1, 2), who employed a zeroth-order boundary-element discretization. The nature of the wake model - a vortex lattice with finite core - was prone to numerical instability as the interacting vortices approached one another. In as much as this behaviour was non-physical and only due to the singularities introduced by the approximation, a number of steps were taken to reduce its effects. The introduction of finite cores and an iterative approach (Ref. 7) dampened the instabilities; however, the solution was not time-accurate. An alternative was sought in the higher-order approximations, wherein the non-physical instabilities would be



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eliminated by replacing the vortex lattice with a first-order doublet layer (i.e., a piece-wise constant vortex layer).

The focus of the research activity in this area has been on the development of a higher-order boundary element method for free-wake analysis. Initially, this entailed the creation of new algorithms for integrating the more complicated source and doublet distributions, thereby determining the aerodynamic coefficients that govern the shape of the flow field. Although the zeroth-order integrals lent easily to analytical expressions, this was not the case for the higher-order integrations. Originally, a fully numerical scheme was employed: this met with limited success, as it failed to exhibit the singular nature of the doublet for control points close to the panel, as well as being extremely costly in terms of computational throughput. In an effort to capture the singularity inherent in the integrals, a combined analytical/numerical integration scheme was developed. In this approach, an analytical integration was performed in one direction, followed by a Gaussian quadrature along the remaining axis (Ref 11). Although this method produced results that converged to the zeroth-order results, the convergence rate of the first order formulation was slower than that for the zeroth-order one.

Thus, a new algorithm was developed that enabled the analytical integration of the doublet and source distributions for any order by a recursive relation relating higher to lower order integrals on a panel tangent to the original hyperbolic paraboloid at a point that minimizes the distance to the control point. The source and doublet integrands on the original hyperboloidal panel may be rewritten as source and doublet integrands on the tangent panel multiplied by a correcting factor. By approximating this factor with a polynomial, the coefficients for the original panel may be computed on the tangent panel (Ref. 12). This method has the merit of analytical integration as well as computational speed (Figs. 1-2).

The second tier in preparation for the free-wake code was to examine the performance of these integration schemes in the fixed-wing steady-state case. A convergence analysis of the zeroth-order and the first-order analytical/numerical methods indicated the similarity of the results as the mesh density was increased. Unfortunately, however, the tangent-element method displayed a singular behavior on the twisted panels as the mesh size was increased. Although this problem was later found to be due to a bug in the code, and corrected, it did govern the initial design of the free-wake code: it was decided to solve the integral equation for the potential using a zeroth-order formulation, with a first-order formulation used only for the free-wake analysis.

More precisely, the free-wake code was designed to take full advantage of the availability of a higher-order analytical integration scheme, while compensating for its deficiencies over the twisted panels. In this respect, the potential is initially computed over the body by the zeroth-order integration; the potential may then be safely computed above and below the surface of the wake using one of the first-order integration schemes. On finite-differencing the potential across the discontinuity (and adding the contribution tangent to the surface), the perturbation velocity of wake may be computed at each time step. As the frame of reference is attached to the body, this computed velocity combined with the boundary conditions on the body determines the new wake geometry.

The free-wake analysis code has been written to simulate both the fixed-wing and helicopter rotor cases. At present, the code is complete. Initially, the cases examined have been for fixed-wing flight conditions. The limitation is not one of software, but analysis: the data sets generated that detail the state of the solution at each time-step are much more amenable in the fixed wing cases due to their smaller size. The current objective is to reproduce the results of Suciu (Ref. 13) - a zeroth-order fixed-wing free-wake analysis and to proceed on to the rotor cases in lieu of these successes.

As mentioned above, the problem with the higher-order tangent element method has been resolved: the scalar potential converges quite quickly to the converged zeroth-order solution (Figs. 3-5). Thus, the free-wake solution may be computed without requiring the zeroth-order integration as an intermediate step. It must be noted that the recursive structure of this algorithm allows one to compute integrals of any order. In addition, the analytical scheme provides a significant increase in computational throughput.

The activity discussed above is the topic of the doctoral thesis of M. J. Downey (Ref. 14), which includes the extension to the free-wake analysis of helicopter rotors as well. The thesis is expected to be completed in the spring of 1990.

4 Poincaré decomposition for viscous flows

A new potential-vorticity decomposition, called the Poincaré Decomposition, has been used as the basis of a numerical implementation of a boundary element method for the analysis of viscous flows past arbitrary bodies. The formulation, applicable to unsteady compressible viscous flows, is based on decomposing the velocity field into two terms, one related to the vorticity (vortical velocity) and one that is irrotational (potential velocity). Unlike the classical Helmholtz decomposition, the new decomposition is such that the vortical velocity is zero over the majority of the irrotational region. Since the efficiency of the boundary integral equation method is related to the extent over which the source terms in the field are non-zero, it is preferable to use a decomposition in which the contribution due to the rotational part of the velocity field is identically zero in the irrotational region. Thus the new decomposition has a considerable advantage over the Helmholtz decomposition, especially for the boundary integral equation solution of compressible viscous flows.

The decomposition was introduced by Morino (Ref. 15). A more detailed presentation of the formulation, including its relation to and its advantages over the classical Helmholtz decomposition, is presented in Morino (Ref. 16; this will appear as chapter of a book that will be published shortly). A paper that emphasizes the relationship between the potential flow formulation and the Poincaré decomposition approach was presented at the International Symposium on Boundary Element Methods held in East Hartford in October 1989 (Morino, Ref. 17).

The focus of the current research is on the implementation of a numerical procedure based on the Poincaré decomposition. It addresses the development of a "proof of concept" algorithm for the time accurate solution of the incompressible, two-dimensional, viscous flow past a body in arbitrary motion. This "proof of concept" algorithm is seen as a first step in the development of a method for the analysis of viscous unsteady three-dimensional compressible flows.

The use of the Poincaré decomposition as the basis of a numerical technique for the analysis of viscous flow problems requires the resolution of a number of issues. The technique developed (Morino and Beauchamp, Ref. 18), employs the boundary element method, to determine the velocity distribution in the field about the body, while the evolution of the vorticity field in time is solved via a finite difference method. One of the key issues in the implementation is the generation of vorticity at the surface of the body. This vorticity generation has been examined by treating the continuous motion of the surface as the limit of a series of relaxation periods separated by discrete velocity impulses. This "surface vorticity generation condition" has been implemented in two distinct ways in the numerical algorithms developed thus far.

In our previously published results (Morino and Beauchamp, Ref. 19) the numerical technique for determining the vorticity generation on the surface was based on the simultaneous solution of the potential and the vortical velocity on the surface. This technique involves

the inversion of a large matrix to determine both the potential and vortical velocity on the surface. The chief drawback of the simultaneous solution technique is that the requirement of a square matrix for the inversion, dictates the number of points on the surface at which the vorticity generation condition must be applied. For symmetric two dimensional flows a square matrix can be easily obtained if the surface vorticity generation condition is evaluated at a set of staggered grid points. In particular, the potential and vortical velocity are found at the collocation points by evaluating the vorticity generation condition at the mid-point between the collocation points. While this technique is useful for two-dimensional symmetric flows, it is not readily applicable to non-symmetric flows and three dimensional flows.

With this in mind, much of our recent work has been directed at the application of an alternative implementation of the surface vorticity generation condition. In this formulation the continuous motion of the surface is considered to be made up of a velocity impulse, during which a surface vorticity layer is generated, followed by a relaxation period during which the vorticity is immediately diffused. This implementation, which will be referred to as the sequential solution method, follows more closely the original surface vorticity generation scheme presented in Morino (Ref. 16).

The sequential solution method has been applied to the analysis of the viscous flow past a flat plate. In developing the technique, however, it was found that an additional numerical step needed to be introduced to the solution process. When the method is implemented as originally described, then as shown in Figure 6, the vorticity distribution along the surface exhibits a high frequency oscillation. This oscillation is initially confined to the leading and trailing edge regions, and it propagates toward the center of the body in time. It was found that the oscillation could be removed by applying the Fast Fourier Transform (FFT) at the end of each time step to remove a numerically induced single high frequency component (at least for constant grid point spacing). Similar results are obtained by locally averaging the solution at adjacent nodes, to obtain the center values and then re-interpolating the nodal values.

The sequential solution technique has been verified by performing a comparative study between it and the results of the simultaneous solution algorithm. The problem examined is the transient-response of a flat plate at zero angle of attack in a viscous flow at a Reynolds number of 1,000. For this comparison, the grid consists of 35 equally spaced elements along the x -axis. There are 5 equally spaced elements extending to a quarter of the chord upstream of the leading edge. On the surface of the plate there are 20 equally spaced elements between the leading and trailing edge. Finally, there are 10 equally spaced elements extending downstream by half the chord length. The grid in the y direction has 5 quadratically spaced elements on either side of the flat plate extending to $y_{max} = \pm 0.1$.

The time increment used for the comparison is $\Delta t = 0.001$. Figures 7 through 9 depict the magnitude of the velocity, as a function of y , at $x = 0.25, 0.5, 0.75$, and 1.0 , for different values of the time corresponding to $t = 0.02, 0.04, 0.06, 0.08$, and 0.1 , respectively. (The free stream velocity is shown by the solid line above the leftmost distribution.) The numerical results obtained by the sequential solution algorithm are plotted as a solid line against the results obtained from the simultaneous solution algorithm, which are represented by the dotted line.¹ The simultaneous solution algorithm results were obtained by running exactly the same grid. The figures illustrate that the two algorithms are in excellent agreement except for a very small difference in the trailing edge region. Figure 10 shows a comparison of the magnitude of the vortical velocity along the line $y = 0$ for the two algorithms at $t = 0.1$. The solid line is the sequential solution algorithm, whereas the dotted line is the simultaneous

¹It should be noted that, although the plotting is done by connecting the points using straight lines, the velocity distribution between the nodes, as obtained from the discretised integral representation is quadratic.

solution algorithm. The two results are identical to within 0.6%. The slight discrepancy can be traced to the fact that the simultaneous solution algorithm yields a larger vorticity peak in the leading-edge region than the sequential solution algorithm. The similarity between the two results is significant because the simultaneous solution algorithm determines the vortical velocity directly, while in the sequential solution algorithm the vortical velocity is derived from the computed vorticity.

The activity discussed above is the topic of the doctoral thesis of P. P. Beauchamp (Ref. 20), which includes the complete details of the formulation and its implementation as well as extensive numerical results. The thesis is expected to be completed by December 14, 1989; the defense was held in October 1989; minor changes are now being included.

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18. L. Morino and P. P. Beauchamp, "A New Potential-Vorticity Decomposition for the Boundary-Element Analysis of Viscous Flows," presented at International Conference on Computational Engineering Science, Atlanta, April 11-15, 1988.
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APPENDIX A

PUBLICATIONS

Theses

H. Affes, "A Boundary-Element Aerodynamic Analysis of Helicopter Rotors," Master Thesis in Mechanical Engineering, Boston University, September 1987.

P. P. Beauchamp, "A Potential-Vorticity Decomposition for the Boundary Integral Equation Analysis of Viscous Flows," Ph. D. Thesis, Graduate School, Division of Engineering and Applied Science, Boston University, Boston, MA, USA (in preparation).

M. J. Downey, "Wake Transport Analysis with Applications to Helicopter and Airplane Aerodynamics," Ph. D. Thesis, Boston University Graduate School, Division of Engineering and Applied Science (in preparation).

Conference Papers

L. Morino, M. Freedman, and K. Tseng, "Unsteady Three-Dimensional Compressible Potential Aerodynamics of Helicopter Rotors in Hover and Forward Flight - A Boundary Element Formulation," American Helicopter Society National Specialist Meeting on Aerodynamics and Aeroacoustics, Arlington, TX, 1987.

L. Morino, B. K. Bharadvaj, M. Freedman, and K. Tseng, "BEM for Wave Equation with Boundary in Arbitrary Motion and Applications to Compressible Potential Aerodynamics of Airplanes and Helicopters," IUTAM Symposium on Advanced Boundary Element Method, San Antonio Texas, April 1987.

L. Morino and M. J. Downey, "A Recursive Algorithm for the Evaluation of Arbitrary Source and Doublet Distribution," International Conference on Computational Engineering Science, Atlanta, April 11-15, 1988.

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L. Morino, "A Unified Approach for Potential and Viscous Flows in Fixed-Wing and Rotary-Wing Aerodynamics," International Symposium on Boundary Element Methods, East Hartford, CT, October 1989.

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Journal Papers

L. Morino and B. K. Bharadvaj, "A Unified Approach for Potential and Viscous Free-Wake Analysis of Helicopter Rotors," Vertica, Vol. 12, No. 1, July 1988.

L. Morino, B. K. Bharadvaj, M. Freedman, and K. Tseng, "Boundary Integral Equation for Wave Equation with Moving Boundary and Applications to Compressible Potential Aerodynamics of Airplanes and Helicopters," "Computational Mechanics," Vol. 4, pp. 231-243, 1989.

Book Chapters

L. Morino, "Helmholtz and Poincaré Vorticity-Potential Decompositions for the Analysis of Unsteady Compressible Viscous Flows," in Eds.: P. K. Banerjee and L. Morino, *Developments in Boundary Element Methods, Volume 6: Nonlinear Problems of Fluid Dynamics*, Elsevier Applied Science Publishers, Barking, UK, 1990.

L. Morino and K. Tseng, "A General Integral Formulation for Unsteady Compressible Potential Flows with Applications to Airplane and Rotors," in Eds.: P. K. Banerjee and L. Morino, *Developments in Boundary Element Methods, Volume 6: Nonlinear Problems of Fluid Dynamics*, Elsevier Applied Science Publishers, Barking, UK, 1990.

APPENDIX B

ABSTRACTS

Theses

H. Affes, "A Boundary-Element Aerodynamic Analysis of Helicopter Rotors," Master Thesis in Mechanical Engineering, Boston University, September 1987.

A numerical iterative scheme for the aerodynamic analysis of helicopter rotors is introduced. This scheme is developed in an attempt to clarify the issue of an unstable phenomenon in the wake geometry which was first observed by Morino and Bharadvaj (1985). The method of solution is based on a boundary integral equation developed by Morino, Kaprielian and Sipic (1983) and refined by Morino and Bharadvaj (1985). The integral equation is discretized by dividing the boundary surface into quadrilateral elements and assuming the unknown terms to be constant within each element. In other words, the formulation used may be classified as zeroth-order direct-approach boundary element method. The wake geometry is obtained, step-by-step, by calculating the location of a wake point at a given time-step from its location at the preceding time-step. The wake model used to facilitate the implementation of this approach is outlined. The description of the iterative scheme used in the time-domain integration scheme is included. The method of evaluating the sectional lift coefficient is outlined. The numerical results are presented and discussed. The concluding remarks and recommendations for future work are also included.

P. P. Beauchamp, "A Potential-Vorticity Decomposition for the Boundary Integral Equation Analysis of Viscous Flows," Ph. D. Thesis, Graduate School, Division of Engineering and Applied Science, Boston University, Boston, MA, USA.

A computational technique is developed based on an exact formulation of the viscous flow problem using a new potential-vorticity decomposition called the Poincaré decomposition. In this decomposition the component of velocity due to the rotational portion of the flow is zero over the majority of the irrotational region. This has considerable advantages over the classical Helmholtz decomposition, especially for the boundary integral equation solution of compressible viscous flows.

This work represents the first step in developing a numerical procedure based on the Poincaré decomposition. It addresses the time accurate solution of the incompressible, two-dimensional, viscous flow past a body in arbitrary motion. The technique developed employs the boundary integral equation method, also called the boundary element method, to determine the velocity distribution in the field about the body, while the evolution of the vorticity field in time is solved using a finite difference method. The complete details of both numerical methods are provided. Also shown is the general solution of the Poincaré decomposition for curvilinear co-ordinates. The generation of vorticity at the surface of the body is examined by treating the continuous motion of the surface as the limit of a series of relaxation periods separated by discrete velocity impulses. This process is examined using three different numerical techniques. The specification of boundary and initial conditions is addressed. The application of the method to steady state problems is outlined.

The results demonstrate that the method can be used for the computation of two-dimensional, incompressible viscous flows and that continued development of the technique is warranted. Numerical results using the technique are presented for flows both with and without solid bodies in the field. The free field solutions (i.e., the case where there is no body) examine the development of an initially Gaussian distribution of vorticity in the field. These results are compared to the exact solution for this problem and are used to validate the finite difference solvers for the vorticity evolution and to ascertain the impact of numerical precision on the solver. The motion of a body in the field is illustrated by the computation of the laminar flow past a flat plate at various conditions. These results are compared with the Blasius solution, for the steady state case, and with other numerical methods when applicable.

M. J. Downey, "Wake Transport Analysis with Applications to Helicopter and Airplane

Aerodynamics," Ph. D. Thesis, Boston University Graduate School, Division of Engineering and Applied Science (in preparation).

Conference Papers

L. Morino, M. Freedman, and K. Tseng, "Unsteady Three-Dimensional Compressible Potential Aerodynamics of Helicopter Rotors in Hover and Forward Flight - A Boundary Element Formulation," American Helicopter Society National Specialist Meeting on Aerodynamics and Aeroacoustics, Arlington, TX, 1987.

A new general boundary-element methodology for the analysis of helicopter rotors in potential compressible flows is presented. The methodology is based on the use of the velocity potential (instead of the more common acceleration potential). The advantages of the velocity potential approach over the acceleration potential approach is discussed in detail. The derivation of the integral equation is outlined, along with the boundary-element algorithm used for the computational implementation. In addition, numerical results for a helicopter rotor in hover are studied in detail, with particular emphasis on the convergence analysis. The numerical results are in excellent agreement with those of Rao and Schatzle.

L. Morino, B. K. Bharadvaj, M. Freedman, and K. Tseng, "BEM for Wave Equation with Boundary in Arbitrary Motion and Applications to Compressible Potential Aerodynamics of Airplanes and Helicopters," IUTAM Symposium on Advanced Boundary Element Method, San Antonio Texas, April 1987.

This work deals with a general boundary-element methodology for the solution of the wave equation around objects moving in arbitrary motion, with applications to unsteady potential compressible aerodynamics of streamlined bodies (such as airplanes and helicopters). The paper includes the derivation of the boundary integral equation for the wave equation, for a frame of reference moving in arbitrary motion (in particular, in translation and in rotation). Also included are (i) a discussion of the formulation for the analysis of the motion of the wake, and (ii) the boundary-element algorithm used for the computational implementation. Validation of the formulation is presented for the cases of airplane wings and helicopter rotors. The test cases fall into two categories: prescribed-wake and free-wake analyses. The validation of the prescribed-wake analysis is presented for compressible flows (subsonic for helicopter rotors, transonic for airplanes). The numerical validation of the free-wake analysis of helicopter rotors is presented for incompressible flows.

L. Morino and M. J. Downey, "A Recursive Algorithm for the Evaluation of Arbitrary Source and Doublet Distribution," International Conference on Computational Engineering Science, Atlanta, April 11-15, 1988.

This paper presents an efficient recursive algorithm for the evaluation of the higher-order coefficients that arise in the application of the boundary-element method to the Laplace equation. This involves the evaluation of source and doublet integrals with arbitrary intensity distributions over surface elements with arbitrary smooth geometry. The surface elements are assumed to be topologically quadrilateral (in the limit, triangular).

L. Morino and P. P. Beauchamp, "A New Potential-Vorticity Decomposition for the Boundary-Element Analysis of Viscous Flows," International Conference on Computational Engineering Science, Atlanta, April 11-15, 1988.

This paper deals with a computational formulation for the analysis of viscous flows about arbitrary bodies. A new decomposition for unsteady incompressible viscous flows is presented. The decomposition is of the type $\mathbf{v} = \nabla\phi + \mathbf{w}$, with $\nabla \times \mathbf{w} = \boldsymbol{\zeta}$, where $\boldsymbol{\zeta}$ is the vorticity. The last equation is solved by direct integration; this yields a particular solution that is equal to zero outside the vortical region. Issues that arise in the use of the decomposition as a computational technique, in particular the vorticity generation at the boundary, are discussed in details.

L. Morino and P. P. Beauchamp, "A Potential-Vorticity Decomposition for the Analysis of Viscous Flows," First Joint Japan/U.S. Symposium on Boundary Element Methods, Tokyo, October 3-6 1988; in Eds.: M. Tanaka and T. A. Cruse, *Boundary Element Methods in Applied Mechanics*, Pergamon Press, 1988.

This paper deals with a methodology for the analysis of viscous flows about arbitrary bodies. The paper is based on a new decomposition for unsteady compressible viscous flows presented in Morino (1988), where theoretical issues, such as vorticity generation and the relationship between viscous and inviscid flows, are emphasized. The use of the formulation as a computational technique was discussed in Morino and Beauchamp (1988), which included an outline of the numerical formulation. Here, the formulation is extended to emphasize issues that arise in the use of the decomposition as a computational technique, in particular, the implementation of the boundary condition on the surface of the body. We also include some numerical results, not previously presented, to demonstrate the feasibility of the methodology.

L. Morino, "A Unified Approach for Potential and Viscous Flows in Fixed-Wing and Rotary-Wing Aerodynamics," International Symposium on Boundary Element Methods, East Hartford, CT, October 1989.

The objective of this paper is to present an elementary introduction of the work of this author and his collaborators in the field of compressible potential aerodynamics and to introduce an extension of the formulation to compressible viscous flows. For the sake of simplicity, details of the formulation are given only for incompressible flows. For compressible flows, the formulations are only outlined.

M. Gennaretti and L. Morino, "A Unified Approach for Aerodynamics and Aeroacoustics of Rotors in Compressible Potential Flows," International Symposium on Boundary Element Methods, East Hartford, CT, October 1989.

The objective of this work is to present a new unified approach to both aerodynamics and aeroacoustics, with applications to hovering helicopter rotors and propellers in subsonic potential flows. This approach consists of solving the velocity-potential equation by a boundary-integral-equation technique and then applying Bernoulli's theorem. The solution for the velocity potential is obtained on the basis of a formulation introduced by Morino, Freedman, Deutsch and Sipic and used by Morino, Bharadvaj, Freedman and Tseng in order to analyze the flow around a hovering helicopter rotor. In this work the numerical implementation has been refined, partially removing the low-Mach-number approximation used by Morino, Bharadvaj, Freedman and Tseng. In addition, a novel approach has been used for the aeroacoustic problem. Specifically, since the late sixties, the work in this field is primarily based on the equations introduced by Ffowcs Williams and Hawkings. These allow one to evaluate the aeroacoustic pressure, once the pressure distribution on the body surface and Lighthill's stress tensor in the field have been evaluated by an independent process (theoretical or experimental). With the present methodology, a single formulation is used to evaluate the velocity potential, on the body and in the field (and then the pressure in the field using Bernoulli's theorem).

F. Mastroddi and L. Morino, "Time and Frequency Domain Aerodynamics for Flutter of Helicopter Rotors in Hover," International Symposium on Boundary Element Methods, East Hartford, CT, October 1989.

The objective of this work is to assess the accuracy of the aerodynamic modeling used in the flutter analysis of helicopter rotors in hover. A boundary-element formulation for unsteady potential incompressible flows is introduced. The results obtained for the aerodynamic forces, for pitch and flap, are compared with those typically used in the flutter analysis of rotors, (which are obtained using a strip-theory extension of Loewy's formulation). The results demonstrate that Loewy's formulation yields results that are considerably different from those obtained with the more accurate model introduced here.

Journal Papers

L. Morino and B. K. Bharadvaj, "A Unified Approach for Potential and Viscous Free-Wake Analysis of Helicopter Rotors," *Vertica*, Vol. 12, No. 1, July 1988.

A unified formulation for the potential and viscous free-wake analysis of helicopter rotors in incompressible, inviscid and viscous, flows is presented. The wake is treated as a vortex layer, with zero thickness for potential flows and finite thickness for viscous flows. The numerical algorithm for the discretization is outlined. Numerical results are in good agreement with the numerical results of Rao and Schatzle and the experimental ones of Landgrebe and of Shivananda.

L. Morino, B. K. Bharadvaj, M. Freedman, and K. Tseng, "Boundary Integral Equation for Wave Equation with Moving Boundary and Applications to Compressible Potential Aerodynamics of Airplanes and Helicopters," *Computational Mechanics*, Vol. 4, pp. 231-243, 1989.

This work presents a general boundary-integral-equation methodology for the solution of the wave equation around objects moving in arbitrary motion, with applications to compressible potential aerodynamics of airplanes and helicopters. The paper includes the derivation of the boundary integral equation for the wave equation, for a frame of reference moving in arbitrary motion (in particular, in translation and in rotation). The formulation is then applied to study unsteady potential compressible aerodynamic flows around streamlined bodies, such as airplanes and helicopters. The formulation is given in terms of the velocity potential, for which an explicit treatment of the wake is required; a discussion of the formulation for the wake transport is included. The advantages of the velocity-potential formulation over the acceleration-potential formulation are discussed. The boundary-element algorithm used for the computational implementation is briefly outlined. Validation of the formulation is presented for airplane wings and helicopter rotors in hover. The test cases fall into two categories: prescribed-wake and free-wake analyses. The validation of the prescribed-wake analysis is presented for compressible flows, subsonic for helicopter rotors, transonic for airplanes. The numerical validation of the free-wake analysis of helicopter rotors is presented for incompressible flows.

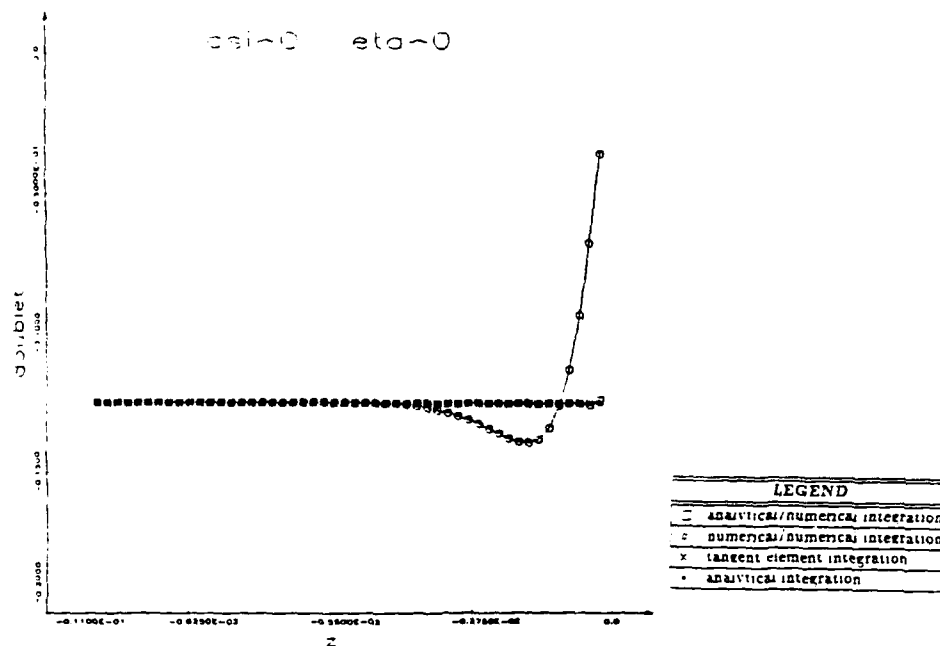
Book Chapters

L. Morino, "Helmholtz and Poincaré Vorticity-Potential Decompositions for the Analysis of Unsteady Compressible Viscous Flows," in Eds.: P. K. Banerjee and L. Morino, *Developments in Boundary Element Methods, Volume 6: Nonlinear Problems of Fluid Dynamics*, Elsevier Applied Science Publishers, Barking, UK, 1990.

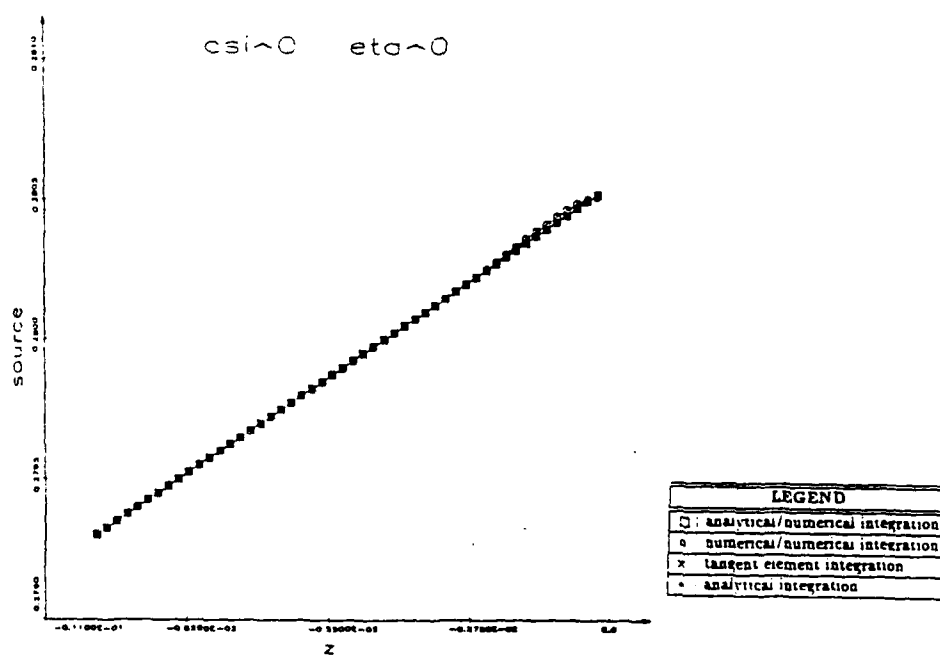
The Helmholtz decomposition for the analysis of unsteady, viscous and inviscid, incompressible flows is reviewed, with particular emphasis on the mechanism of the vorticity generation. Computational methods based on this decomposition are presented, for both inviscid and viscous flows. It is shown that the solutions for viscous attached high-Reynolds-number flows and for inviscid flows are close to each other, provided that the Kutta-Joukowski trailing-edge condition is satisfied for inviscid flows. The incompressible-flow formulation is then extended to compressible flows. It is shown that the Helmholtz decomposition is not convenient for the boundary-element analysis of compressible flows, because the rotational source terms are different from zero in the irrotational region. A new decomposition, called the Poincaré decomposition, is introduced, for which the rotational source terms are equal to zero in most of the irrotational region. This makes the decomposition appealing for boundary-element solution of compressible viscous flows. Simple numerical results, for incompressible two-dimensional flows, obtained using a numerical formulation based on the Poincaré decomposition are presented in order to demonstrate the feasibility of the method.

L. Morino and K. Tseng, "A General Integral Formulation for Unsteady Compressible Potential Flows with Applications to Airplane and Rotors," in Eds.: P. K. Banerjee and L. Morino, *Developments in Boundary Element Methods, Volume 6: Nonlinear Problems of Fluid Dynamics*, Elsevier Applied Science Publishers, Barking, UK, 1990.

This work presents a general boundary-integral-equation methodology for the solution of potential compressible flows around objects moving in arbitrary motion, with applications to aerodynamics of airplanes, helicopter rotors, and propellers. The formulation is given in terms of the velocity potential, for which an explicit treatment of the wake is required; a discussion of the formulation for the wake transport is included. The direct approach is used, i.e., the unknown is the velocity potential itself. The paper includes a review of the literature in the field, the mathematical foundations of potential flows, an introductory section on incompressible flows, and a formulation for compressible flows with the derivation of a boundary integral equation for the velocity potential equation in a frame of reference moving in arbitrary motion. The formulation is then specialized for the particular cases of uniform translation (airplanes), uniform rotation (helicopter rotors in hover), and uniform helicoidal motion (propellers). Numerical results for several cases are included.

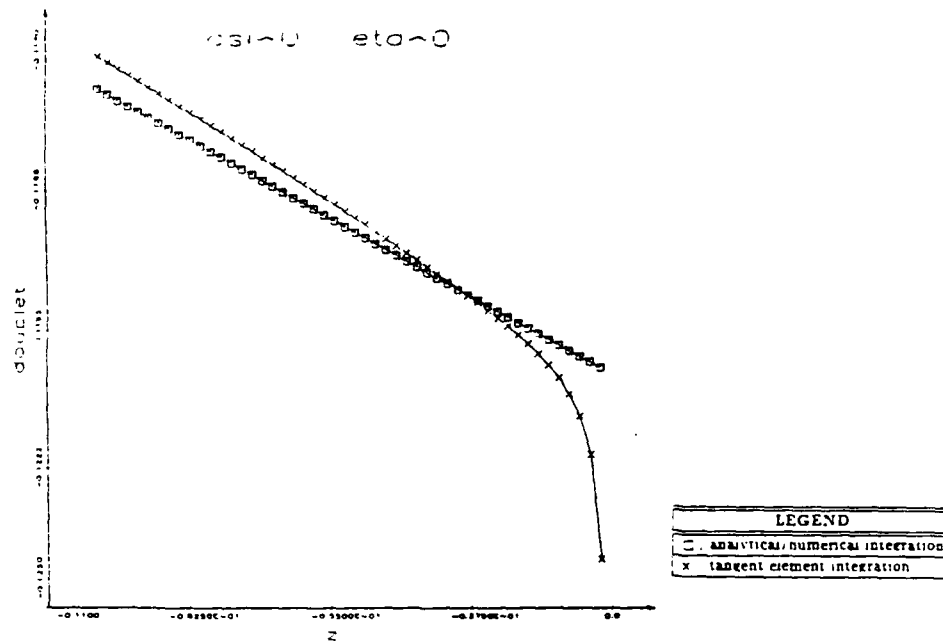


(a) Doublets

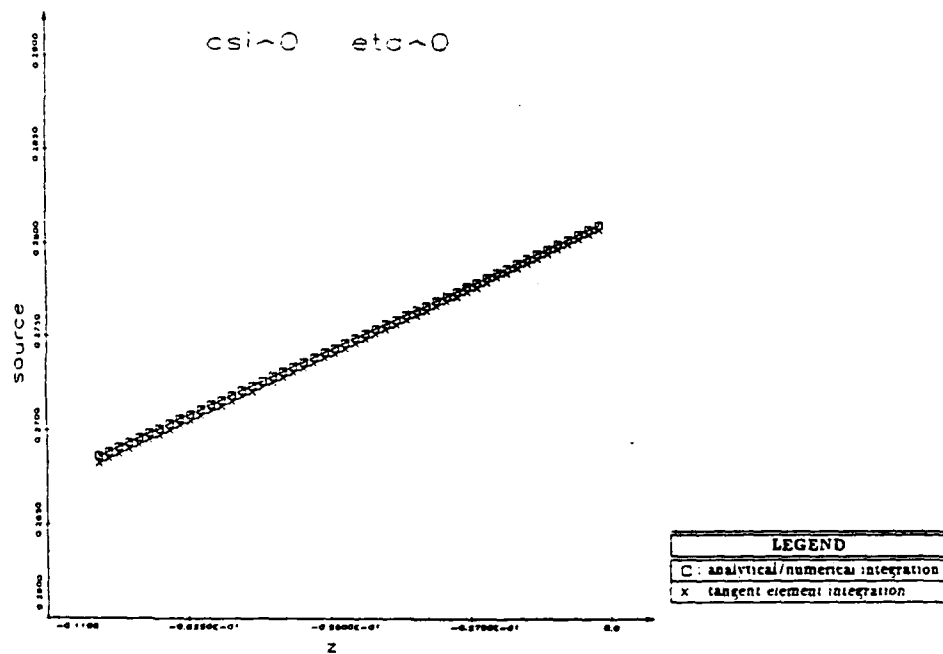


(b) Sources

Figure 1: A comparison of the integration schemes on a 2x2 flat panel centered at the origin (the analytical solution is tractable for this geometry), as the control point moves normal to plane at the $(-1.0, -1.0)$ vertex.



(a) Doublets



(b) Sources

Figure 2: A comparison of the two approximate analytical and numerical schemes for the curved panel geometry: the $(-1.0, -1.0)$ vertex has been raised to 0.2 in the normal direction.

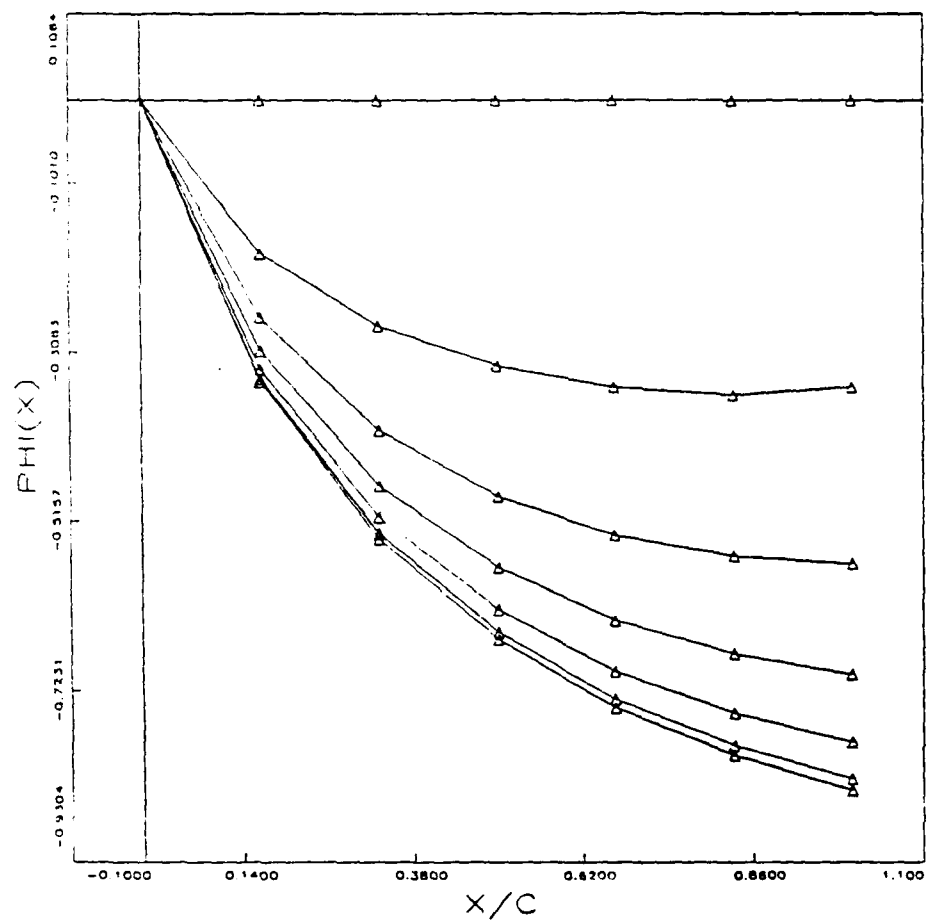


Figure 3: First-order fixed-wing fixed-wake solution for lifting body: using analytical numerical integration on biconvex airfoil. 6x6 element case.

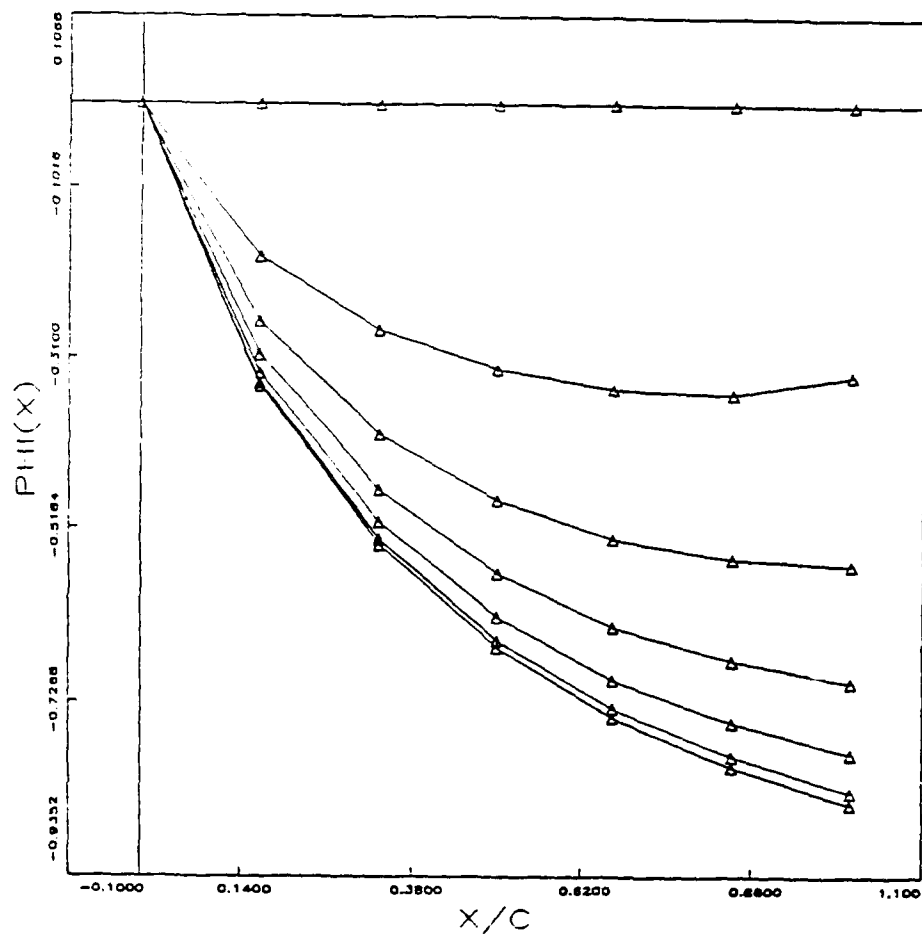


Figure 4: First-order fixed-wing fixed-wake solution for lifting body: using tangent element integration on biconvex airfoil. 6x6 element case.

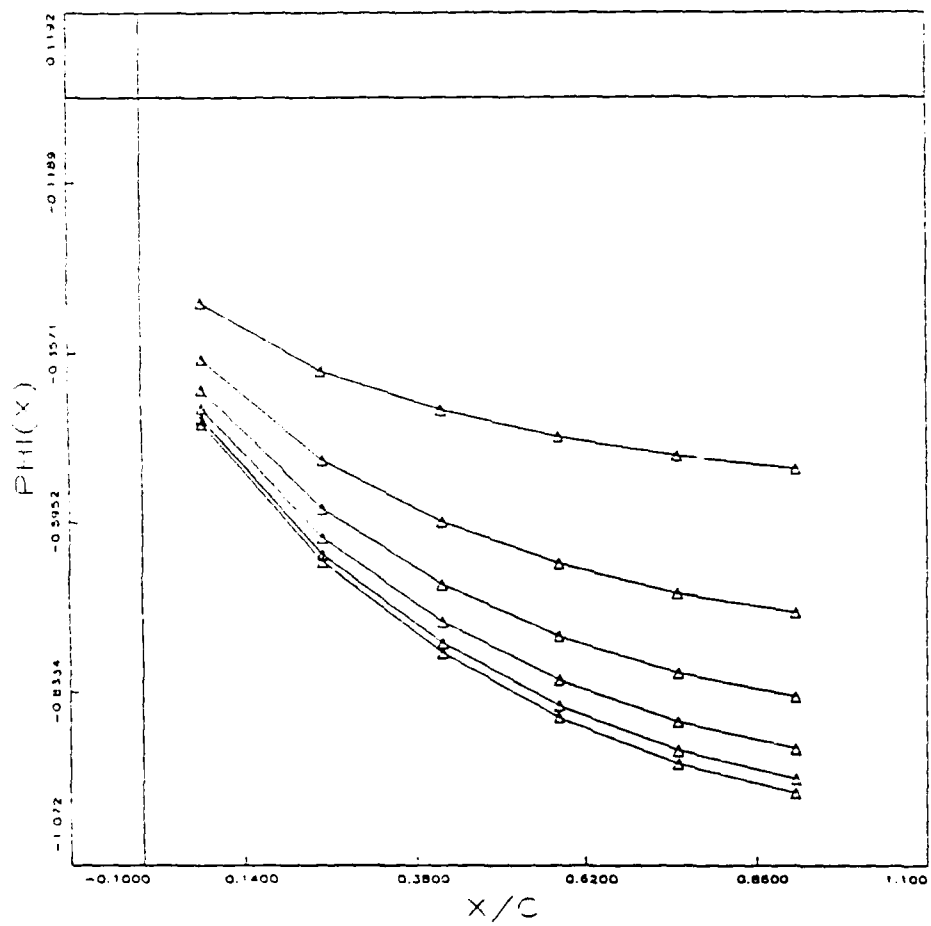


Figure 5: Zeroth-order fixed-wing fixed-wake solution for lifting body: biconvex airfoil. 6x6 element case.

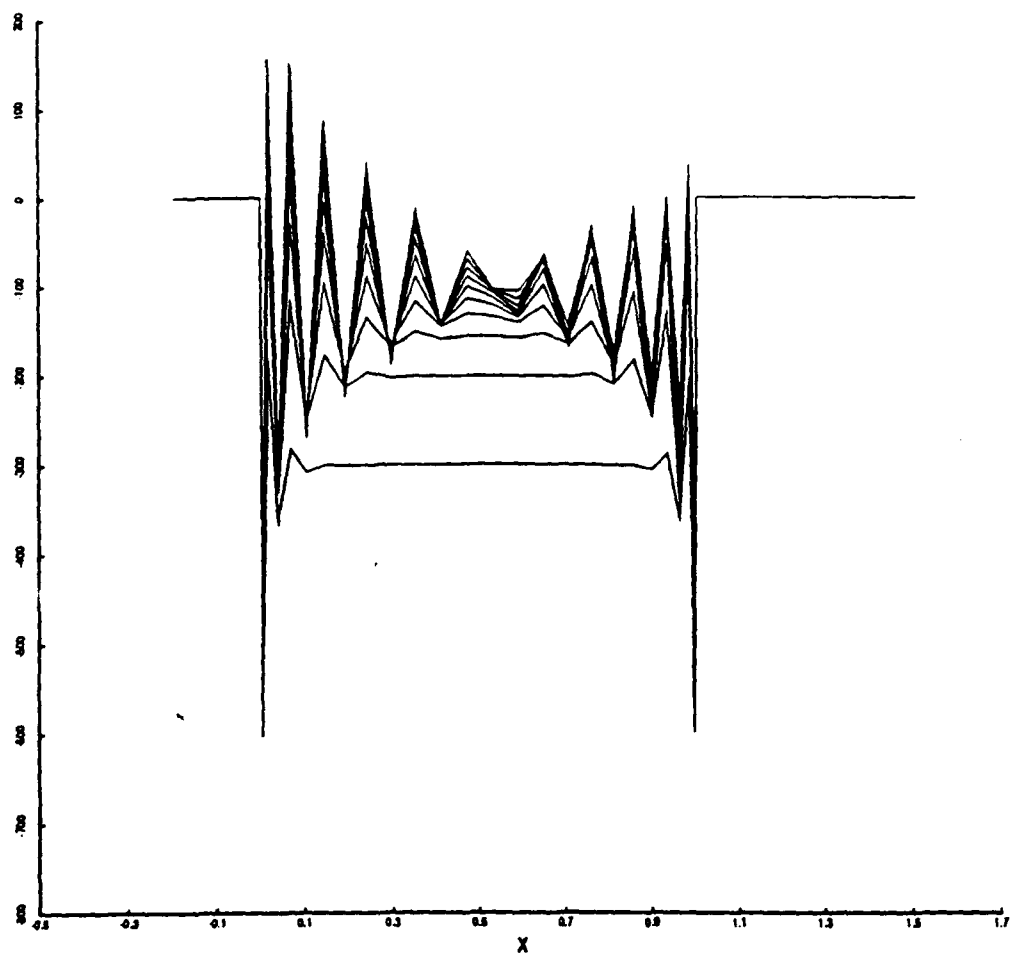


Figure 6: Oscilations on the original implementation of the sequential solution method.

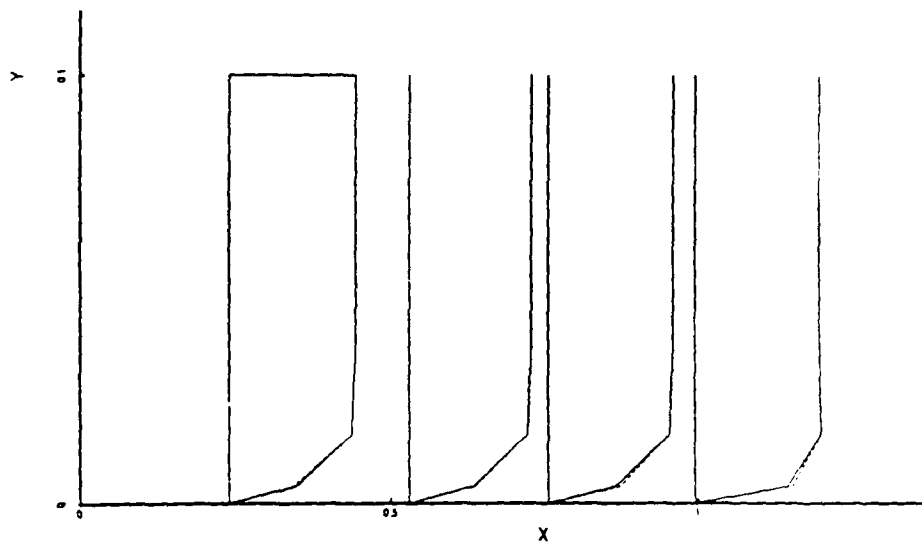


Figure 6 Velocity distribution $|v|$ at $x = .25, .5, .75$ and 1.0 , at $t = .02$; Non-symmetric algorithm (solid); Comparison with the symmetric algorithm(dotted).

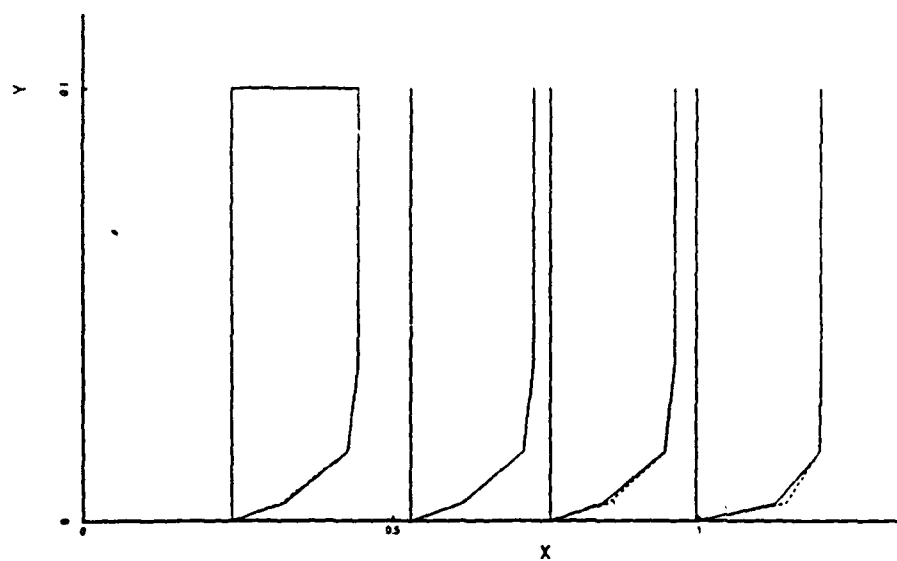


Figure 7 : Velocity distribution $|v|$ at $x = .25, .5, .75$ and 1.0 , at $t = .04$; Non-symmetric algorithm (solid); Comparison with the symmetric algorithm(dotted).

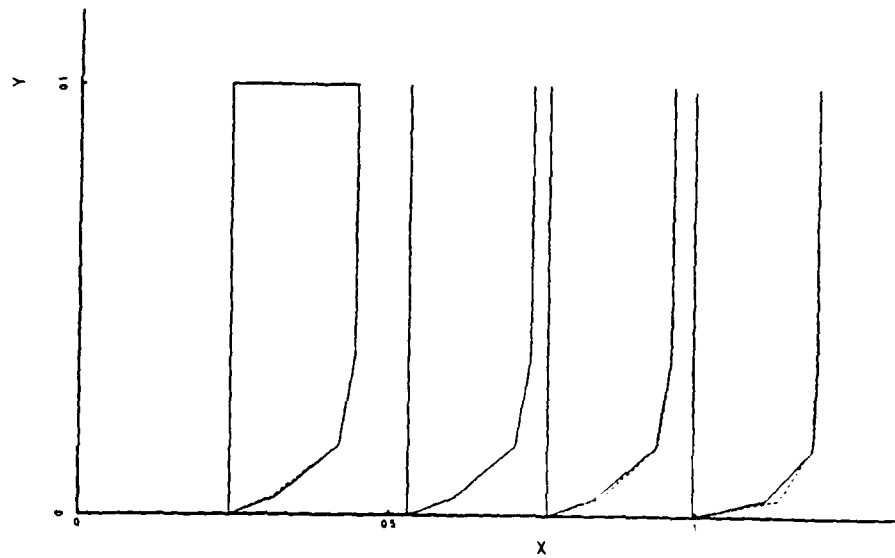


Figure 7 Velocity distribution $|v|$ at $x = .25, .5, .75$ and 1.0 , at $t = .06$; Non-symmetric algorithm (solid); Comparison with the symmetric algorithm(dotted).

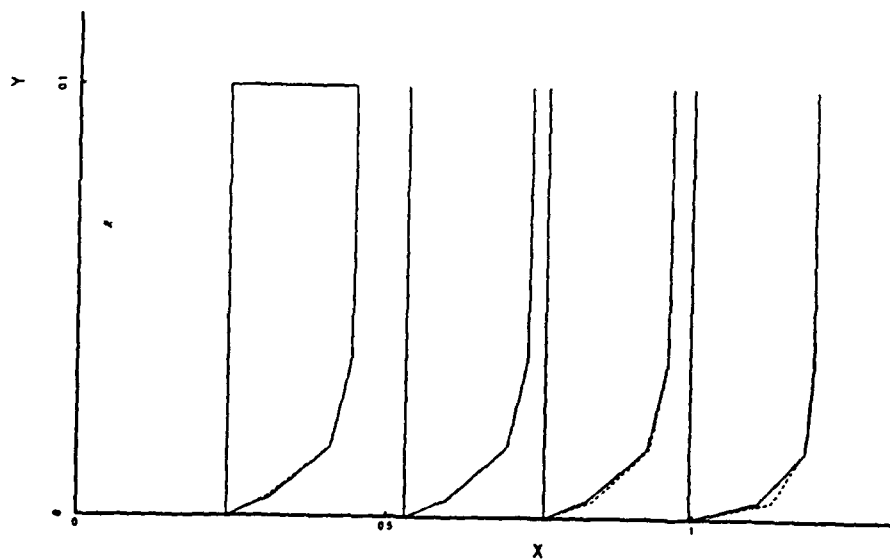


Figure 8 : Velocity distribution $|v|$ at $x = .25, .5, .75$ and 1.0 , at $t = .08$; Non-symmetric algorithm (solid); Comparison with the symmetric algorithm(dotted).

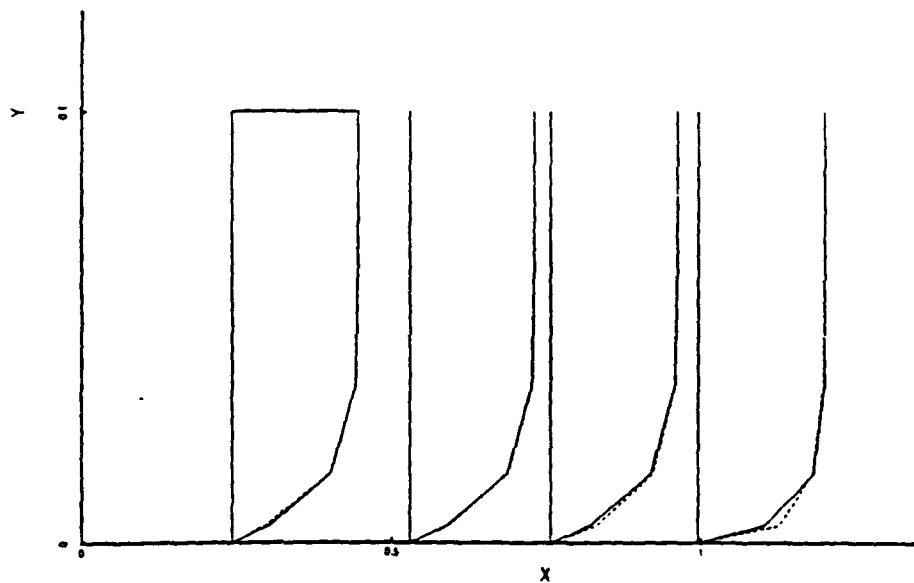


Figure 9 : Velocity distribution $|v|$ at $x = .25, .5, .75$ and 1.0 , at $t = .1$; Non-symmetric algorithm (solid); Comparison with the symmetric algorithm(dotted).

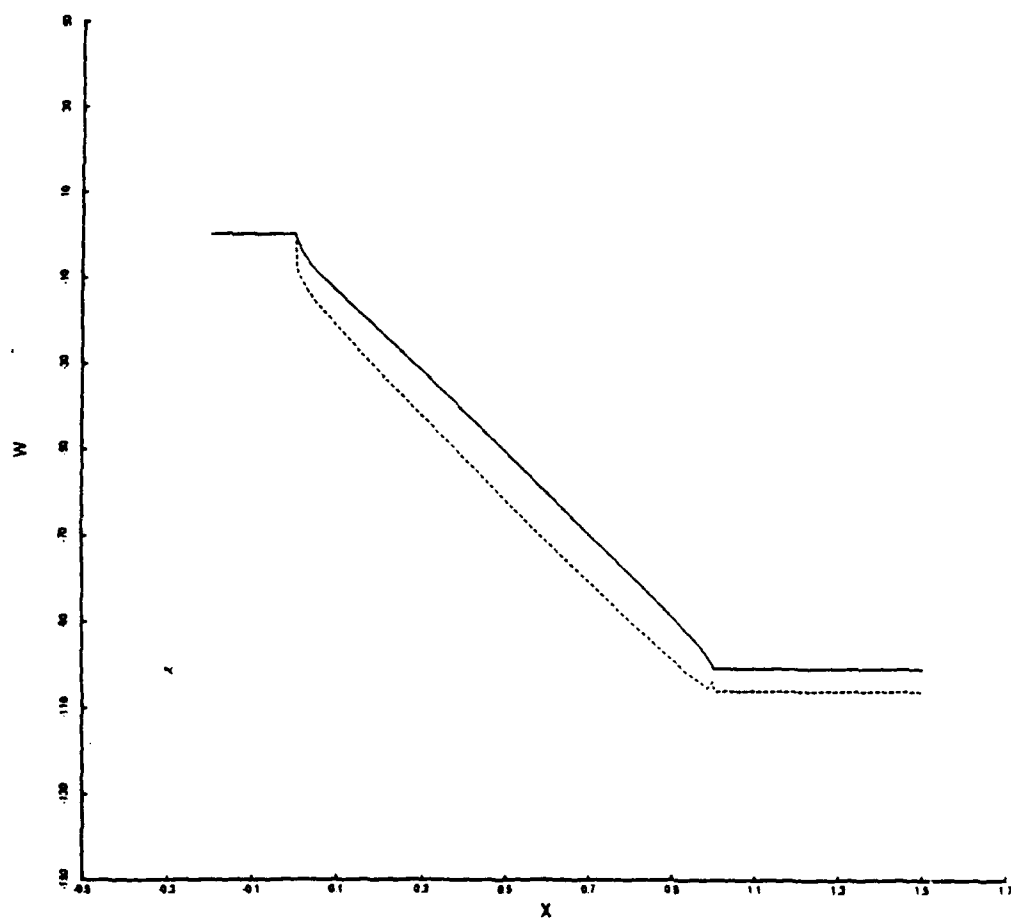


Figure 10.7: Comparison of the vortical velocity at $y = 0$ for $t = .1$ between the Non-symmetric algorithm (solid) and symmetric algorithm(dotted).